# Assignment #6

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**Problem 1**

# Solution to Definite Integrals

## Part (a)

Integral: ∫₄⁹ 3√x dx  
  
Solution:  
We know:  
∫√x dx = (2/3)x^(3/2)  
Thus:  
∫₄⁹ 3√x dx = 3 × (2/3)x^(3/2) = 2x^(3/2)  
  
Apply the limits:  
∫₄⁹ 3√x dx = [2x^(3/2)]₄⁹ = 2(9^(3/2)) - 2(4^(3/2))  
= 2(27) - 2(8) = 54 - 16 = 38  
  
Result: 38

## Part (b)

Integral: ∫₁ᵉ ln(x) dx  
  
Solution:  
Using integration by parts:  
∫ln(x) dx = xln(x) - x  
  
Apply the limits:  
∫₁ᵉ ln(x) dx = [xln(x) - x]₁ᵉ  
= [e ln(e) - e] - [1 ln(1) - 1]  
= [e(1) - e] - [0 - 1]  
= e - e + 1 = 1  
  
Result: 1

## Part (c)

Integral: ∫₀¹ cos⁻¹(x) dx  
  
Solution:  
Using the formula:  
∫cos⁻¹(x) dx = xcos⁻¹(x) - √(1 - x²)  
  
Apply the limits:  
At x = 1:  
1 cos⁻¹(1) - √(1 - 1²) = 1(0) - 0 = 0  
At x = 0:  
0 cos⁻¹(0) - √(1 - 0²) = 0 - 1 = -1  
  
Thus:  
∫₀¹ cos⁻¹(x) dx = 0 - (-1) = 1  
  
Result: 1

## Part (d)

Integral: ∫₋₁¹ πcos(πx/2) dx  
  
Solution:  
Factor out π:  
∫₋₁¹ πcos(πx/2) dx = π ∫₋₁¹ cos(πx/2) dx  
  
Since cos(πx/2) is even:  
∫₋₁¹ cos(πx/2) dx = 2 ∫₀¹ cos(πx/2) dx  
  
We know:  
∫cos(kx) dx = (1/k)sin(kx)  
  
Here, k = π/2:  
∫₀¹ cos(πx/2) dx = [2/π sin(πx/2)]₀¹  
= (2/π)(sin(π/2)) - (2/π)(sin(0))  
= (2/π)(1) - 0 = 2/π  
  
Back to the original integral:  
∫₋₁¹ πcos(πx/2) dx = π × 2 × (2/π) = 4  
  
Result: 4

## Final Results

1. Part (a): 38  
2. Part (b): 1  
3. Part (c): 1  
4. Part (d): 4

**Problem 2**

# Detailed Solution to Indefinite Integrals

## Part (a): ∫ x² cos(x³) dx

Step 1: Use substitution:  
Let u = x³, so that:  
du = 3x² dx, or equivalently, du/3 = x² dx  
  
Step 2: Rewrite the integral in terms of u:  
∫ x² cos(x³) dx = ∫ cos(u) (du/3) = (1/3) ∫ cos(u) du  
  
Step 3: Solve the integral:  
We know that ∫ cos(u) du = sin(u)  
Thus, the integral becomes:  
(1/3) ∫ cos(u) du = (1/3) sin(u) + C  
  
Step 4: Substitute back u = x³:  
The solution is:  
(1/3) sin(x³) + C  
  
**Final Result: (1/3) sin(x³) + C**

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| --- | --- | --- |
| **x** | **f(x) = x^2 \* cos(x^3)** |  |
| -2 | -0.582 |  |
| -1.71429 | 0.939802 |  |
| -1.42857 | -1.98886 |  |
| -1.14286 | 0.101885 |  |
| -0.85714 | 0.593766 |  |
| -0.57143 | 0.320863 |  |
| -0.28571 | 0.08161 |  |
| 0 | 0 |  |
| 0.285714 | 0.08161 |  |
| 0.571429 | 0.320863 |  |
| 0.857143 | 0.593766 |  |
| 1.142857 | 0.101885 |  |
| 1.428571 | -1.98886 |  |
| 1.714286 | 0.939802 |  |
| 2 | -0.582 |  |

## Part (b): ∫ (cos(3t) / (1 + sin(3t))) dt

Step 1: Use substitution:  
Let u = 1 + sin(3t), so that:  
du = 3cos(3t) dt, or equivalently, du/3 = cos(3t) dt  
  
Step 2: Rewrite the integral in terms of u:  
∫ (cos(3t) / (1 + sin(3t))) dt = ∫ (1/u) (du/3) = (1/3) ∫ (1/u) du  
  
Step 3: Solve the integral:  
We know that ∫ (1/u) du = ln|u|  
Thus, the integral becomes:  
(1/3) ∫ (1/u) du = (1/3) ln|u| + C  
  
Step 4: Substitute back u = 1 + sin(3t):  
The solution is:  
(1/3) ln|1 + sin(3t)| + C  
  
Final Result: (1/3) ln|1 + sin(3t)| + C

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| --- | --- | --- |
| **t** | **f(t) = cos(3t) / (1 + sin(3t))** |  |
| -2 | 0.750476 |  |
| -1.71429 | 0.21862 |  |
| -1.42857 | -0.21663 |  |
| -1.14286 | -0.74751 |  |
| -0.85714 | -1.82911 |  |
| -0.57143 | -13.9144 |  |
| -0.28571 | 2.682516 |  |
| 0 | 1 |  |
| 0.285714 | 0.372784 |  |
| 0.571429 | -0.07187 |  |
| 0.857143 | -0.54671 |  |
| 1.142857 | -1.33777 |  |
| 1.428571 | -4.61608 |  |
| 1.714286 | 4.574137 |  |
| 2 | 1.332488 |  |

**Problem 3**

## 1. Define and Graph the Function

The elevation of the path is given by the function:  
f(x) = x³ - 5x² + 30, where x measures horizontal distance. We compute values of f(x) for 15 evenly spaced points in the range [0, 4] and plot the graph.  
  
Values of f(x) at these points have been saved in an Excel file for visualization.

## 2. Find the Average Value of the Function

The average value of a function f(x) over the interval [a, b] is given by:  
Average Value = (1 / (b - a)) ∫ₐᵇ f(x) dx  
  
For this function:  
f(x) = x³ - 5x² + 30, and the interval is [0, 4].

Step 1: Write the formula for the average value:  
Average Value = (1 / (4 - 0)) ∫₀⁴ (x³ - 5x² + 30) dx  
  
Step 2: Compute the integral:  
∫(x³ - 5x² + 30) dx = (1/4)x⁴ - (5/3)x³ + 30x  
Evaluating this from 0 to 4:  
[(1/4)(4⁴) - (5/3)(4³) + 30(4)] - [(1/4)(0⁴) - (5/3)(0³) + 30(0)]

Step 3: Simplify:  
[(1/4)(256) - (5/3)(64) + 120] - [0] = 64 - 106.67 + 120 = 77.33  
  
Step 4: Divide by (4 - 0):  
Average Value = 77.33 / 4 = 19.33

Final Result: The average value of the elevation function is approximately 19.33.

## Graph and Data

The graph data for f(x) has been saved in an Excel file, which can be used for visualization. The Excel file contains 15 evenly spaced points in the range [0, 4] and the corresponding values of f(x).

|  |  |
| --- | --- |
| **x** | **f(x)** |
| 0 | 30 |
| 0.285714 | 29.61516 |
| 0.571429 | 28.55394 |
| 0.857143 | 26.95627 |
| 1.142857 | 24.9621 |
| 1.428571 | 22.71137 |
| 1.714286 | 20.34402 |
| 2 | 18 |
| 2.285714 | 15.81924 |
| 2.571429 | 13.94169 |
| 2.857143 | 12.50729 |
| 3.142857 | 11.65598 |
| 3.428571 | 11.5277 |
| 3.714286 | 12.26239 |
| 4 | 14 |